## AP CALCULUS AB 2022-2023 SUMMER ASSIGNMENT DUE: First day of class

There are certain skills that have been taught to you over the previous years that are necessary to know when coming into AP Calculus. If you do not have these skills, you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for you when you are stumbled by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

AP Calculus AB is a fast-paced course that is taught at the college level. There are a lot of materials in the curriculum that must be covered before the AP exam in May. Therefore, we cannot spend a lot of time re-teaching prerequisite skills. Therefore, you have this packet. Spend time with it and make sure you are clear on everything covered in this packet.

This assignment will be collected and graded for quarter 1 grade. Be sure to show all appropriate work to support your answers. Please DON'T wait until the last minute because these problems do take time. The goal of the assignment is for you to review if not relearn materials that you are expected to know prior to Calculus. You will be required to do all work algebraically without calculator in Calculus. Please don't rely on your calculator to do all the work for you.

Good Luck!
1.Solve each equation below for y. Show work that leads to your answer.
a) $x^{2} y-10 y=4$
b) $\ln (3-y)=k t+4$
c) $\sqrt[3]{2 y+1}=x-1$
d) $-3 e^{2 y}=x+2$
2. Find the slope of the line.
a) $x=-3$
b) $y=0$
c) $-2 x-3 y=2$
d) $A(-2,1)$ and $B(5,-1)$
3. Write an equation in point-slope form for the line that passes through $(1,-4)$ and
a) Parallel to $-3 x+4 y=4$
b) Perpendicular (normal) to $2 x-3 y=4$
4. Factor each expression below completely. No decimal/fraction.
a) $4 x^{4}-81$
b) $5 x^{2}-x-6$
c) $216 x^{3}+27$
d) $8 x^{3}-1$
5. Solve each equation below. Check for extraneous solution. Leave exact answer if possible. Show work that lead to your answer.
a) $\sqrt{x^{2}-9}=x-1$
b) $5^{x}=9$
c) $2 x^{2}-(x-3)(x+2)=12$
d) $\frac{2 x-7}{x+1}=\frac{2 x}{x+4}$
6. Simplify each expression below.
a) $\frac{2 x^{3}}{y^{-4}} \cdot \frac{y^{2}}{3 x^{7}}$
b) $\frac{x-4}{4-x}$
c) $\frac{x^{2}-4 x-5}{x^{2}+2 x+1}$
d) $\frac{\frac{2}{x}-\frac{x-1}{x+1}}{\frac{1}{x}}$
e) $\frac{4 x^{3 / 2} y^{-1 / 4}}{8 x^{1 / 4} y^{2 / 3}}$
7. Use the function $f(x)=\left\{\begin{array}{cc}4-x^{2} & x<1 \\ 1.5 x+1.5 & 1 \leq x \leq 3 \\ x+3 & x>3\end{array}\right.$ to answer the following questions.
a) Find f(1.5)
b) Find $f(-1)$
c) Find f(10)
d) Graph the function $f(x)$

8. If $f(x)=|2 x-4|$, then write $\mathrm{f}(\mathrm{x})$ as a piece-wise function.
9. Given $f(x)=-2 x+1$ and $g(x)=x^{2}+x$. Use these functions to answer the following questions.
a) $f(-2)$
b) $f(g(-1))$
c) Find the inverse of $f(x)$.
e) Find $\frac{g(3+h)-g(3)}{h}$. What is the meaning of the answer?
10. Given $h(x)=x^{3}-4 x-1$
a) [Calculator] Find the zeros of $h(x)$.
b) [Calculator] Find the relative minimum and relative maximum points of $h(x)$.
c) [Calculator] Find all intervals where $\mathrm{h}(\mathrm{x})$ is increasing and where it's decreasing. Use interval notation.
d) Find all inflection points of $h(x)$.
e) On what interval is $h(x)$ concave up? Is concave down? Use interval notation.
11. Find any hole, vertical asymptote, and horizontal asymptote of the following functions if possible. Graph each function.
a) $f(x)=\frac{x^{2}-1}{x^{2}-3 x-4}$
b) $g(x)=2^{x-1}-2$
12. Rationalize the numerator of $\frac{\sqrt{a+h}-\sqrt{a}}{h}$
13. Given $y=\ln x$
a) Find the value of $\ln (1)$ and $\ln (e)$.
b) Find the domain and range of the function.
14. Use graphing calculator to find all intersections points of $f(x)=1+3 x^{2}+3 x^{4}$ and $g(x)=2 e^{x}+2$.
15. Write the equation (not area or circumference formula) of the circle with radius 3 and center at the origin. Graph the function.
16. Use composition of functions to show that $f(x)=4 x-10$ and $g(x)=\frac{x+10}{4}$ are inverses.

## 17. For each question below

## - Write the equations that model the problem

- Solve the equations.


## Show work that lead to your answer.

a) A 13 -foot-long ladder leans on a wall. The bottom of the ladder is 5 feet from the wall. If the bottom is pulled out 3.5 feet farther from the wall, how far does the top of the ladder move down the wall?
b) A rectangle is twice as wide as it is high. If it has an area of 24.5 square inches, what are its dimensions?
c) A triangle has an area of 100 square inches, and its height is two-thirds of its base. What are the base and height of the triangle?
d) A storage locker in the shape of a rectangular prism with a square floor has volume of 70,000 cubic inches. If the surface area of the four walls and the top is 10,000 square inches, what are the dimensions of the locker?
e) Two cars start moving from the same point. One travels south at $100 \mathrm{~km} / \mathrm{hour}$, the other travels west at $50 \mathrm{~km} /$ hour. How far apart are they 3.5 hours later?
f) A kite is 100 m above the ground. If there are 200 m of string out, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight)
18. Find the ratio of the area inside the square but outside the circle to the area of the square of the picture below.

19. Given the triangle below, express $\mathbf{x}$ in terms of the other variables (use similar figures concept to write the ratios)

20. Determine if the function is even, odd, or neither. Justify your answer.
a) $y=-3 x^{4}-2 x-3$
b) $y=\sin (x)$
c) $y=\cos (x)$
d) $y=\frac{5 x}{x^{2}-9}$
21. Evaluate and give exact value without calculator.
a) $\sin \left(\frac{2 \pi}{3}\right)$
b) $\cos \left(\frac{-\pi}{4}\right)$
c) $\tan \left(\frac{\pi}{6}\right)$
d) $\sin ^{-1}\left(\frac{-1}{2}\right)$
e) $\cos ^{-1}\left(\frac{\sqrt{3}}{2}\right)$
f) $\tan ^{-1}(-1)$
22. Sketch/Graph each parent function below and state its domain and range. Be sure to label any x-intercept(s) and y-intercept.
a) $y=e^{x}$
b) $y=\sqrt{x}$
c) $y=\sin x$
d) $y=\cos x$
23. Use trigonometric identities to $\operatorname{simplify}(\sin x-\cos x)^{2}-\sin (2 x)+\cos (2 x)$.
24. Use inverse trigonometric to find all solutions for each trigonometric equation below. Give exact solutions.
a) $\sin x=\frac{1}{2}$
b) $\cos 2 x=\frac{\sqrt{2}}{2}$
c) $\sqrt{3} \tan x=-1$
25. a) What does it mean for any function to have a limit at a particular $x$ value?
b) What does it mean for any function to be continuous at a particular $x$ value?
26. Evaluate each limit algebraically. Show work that lead to the answer!
a) $\lim _{x \rightarrow-2} 3 x^{4}$
b) $\lim _{x \rightarrow-1} x^{3}+3 x^{2}-2 x-17$
c) $\lim _{x \rightarrow-2} \frac{x^{2}-4}{x+2}$
d) $\lim _{x \rightarrow 2} \frac{x^{2}+2 x-3}{x^{2}-9}$
e) $\lim _{h \rightarrow 0} \frac{(3+h)^{2}-9}{h}$
f) $\lim _{x \rightarrow 5} \frac{\sqrt{x-2}-\sqrt{3}}{x-5}$
g) $\lim _{x \rightarrow 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x}$
h) $\lim _{x \rightarrow 0} \frac{3 \sin x}{4 x}$
27. Use one-sided limit (left and right) to explain why the $\lim _{x \rightarrow 3} \frac{1}{x-3}$ does not exist.
28. Does $\lim _{x \rightarrow-\frac{3}{2}} \frac{|2 x+3|}{2 x+3}$ exist? Justify your answer.
29. Given $g(x)= \begin{cases}4-x^{2} & x \leq 2 \\ x-1 & x>2\end{cases}$
a) Find $\lim g(x)$
$x \rightarrow 2^{-}$
b) Find $\lim g(x)$

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x \rightarrow 2^{+}
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c) Does $\lim _{x \rightarrow 2} g(x)$ exist? Justify your answer.
d) Is $g(x)$ continuous at $x=2$ ? Justify your answer.
30. Limit x approaches infinity and negative infinity.
a) Find $\lim _{x \rightarrow \infty} \frac{e^{x}}{4 x^{3}}$
b) $\lim _{x \rightarrow-\infty} \frac{16 x^{3}-7 x-5}{5 x^{2}+7 x^{3}-5 x}$
31. Find the average rate of change of the function $f(x)=\sqrt{x+2}$ over the interval $[-1,2]$.
32. The graph below shows the number of tuberculosis deaths in the United States from 1989 to 1998.


Find the average of change of the number of tuberculosis deaths between 1993 to 1994.
33. Use difference quotient and limit to find the instantaneous rate of change of $y=-2 x^{2}+3 x+2$ at $\mathrm{x}=2$.
34. Define the followings and give an example for each.
a) Secant line
b) Tangent line
c) Normal Line
d) Average rate of change
e) Instantaneous rate of change

